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A Unique Synthesis Method of Transformerless Active RC Networks

by

S. K. Mitra

Series No. 60, Issue No. 411
G-12142
October 12, 1961

ELECTRONICS RESEARCH LABORATORY

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ACKNOWLEDGMENT

The author wishes to thank Professor E. S. Kuh for his encouragement and helpful suggestions. Thanks are also due to Messers. J. D. Patterson and R. A. Rohrer for useful discussions.

ABSTRACT

A unique decomposition of active RC driving-point impedance functions is presented, which has been obtained by considering the driving-point synthesis problem in terms of the reflection coefficient. Application of the decomposition has been shown to guarantee the realization of the driving-point impedance in Kinariwala's cascade configurations and Sandberg's special configurations, each containing one negative impedance converter. The method imposes no restriction on the impedance function, except that it has only to be a real rational function. The decomposition technique can be easily programmed on a digital computer.

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I. INTRODUCTION

Synthesis of active RC networks has received considerable attention in recent years because of the advantages these networks possess compared to other types of networks. A one-port active RC network can be considered to consist of a three-port passive RC network (the so-called associated[1] RC network) to two ports of which is connected an active device. Usually, the three-port RC network is a combination of several one-port and/or two-port RC networks. In general, the steps followed in most of the existing synthesis techniques are:

Step 1 - Decomposition and partitioning of the given function,

Step 2 - Identification of the associated RC network (or networks) parameters,

Step 3 - Synthesis of the associated network.

Although several active RC configurations will be found in the literature, there exists no straightforward decomposition and partitioning method which would result in unique functions characterizing the associated RC network.

The purpose of this paper is twofold. First it presents a unique decomposition of active RC driving-point impedance functions. Second, it shows that the suggested decomposition can be used to realize impedance functions in existing active RC configurations, where the associated networks are now easily obtained. In a sense, this paper proves in a very simple and straightforward way the following:

Theorem: Any real rational function of s, the complex frequency variable, can always be synthesized as the driving-point impedance of an active RC transformerless network containing a single negative impedance converter and associated RC networks which are characterized by unique functions.

The basic idea behind it is to obtain an equivalent active LC driving-point impedance from the given function by an RC-LC transformation. Then the reflection coefficient corresponding to this

equivalent LC impedance is obtained. It is shown that the reflection coefficient can be expressed as a product of two reflection coefficients, one corresponding to the passive portion and the other representing the active portion. Following Kelly's approach,[4] the desired decomposition is obtained upon inverse LC-RC transformation. The suggested decomposition can be programmed on a digital computer.

II. DEVELOPMENT OF THE DECOMPOSITION

A. The Concept of Reflection Coefficient

Consider an active RC driving-point impedance $Z(s) = K\,N(s)/D(s)$ where $Z(s)$ is restricted to be a real rational function of s and the leading coefficients of $N(s)$ and $D(s)$ are assumed to be unity. Using the RC-LC transformation,[5] we can obtain an equivalent active LC driving-point impedance $\hat{Z}(s)$, which is defined as

$$\hat{Z}(s) = s\,Z(s^2) = K\frac{s\,N(s^2)}{D(s^2)} \qquad (1)$$

The reflection coefficient $\hat{\rho}(s)$ corresponding to $\hat{Z}(s)$ is given as

$$\hat{\rho}(s) = \frac{\frac{E_{in}(s)}{2} - E_o(s)}{\frac{E_{in}(s)}{2}} = \frac{1 - \hat{Z}(s)/R_{in}}{1 + \hat{Z}(s)/R_{in}} \qquad (2)$$

according to the notations of Fig. 1. Substituting (1) in (2), we obtain

$$\hat{\rho}(s) = \frac{D(s^2) - s\,N(s^2)}{D(s^2) + s\,N(s^2)} = \frac{Q(s)}{Q(-s)} \qquad (3)$$

where the impedance level has been normalized with respect to R_{in} equal to K.* Now, $D(s^2)$ is an even function and $s\,N(s^2)$ is an odd function. As a result, it is clear from (3) that to each zero of $\hat{\rho}(s)$ in the left-half s-plane there corresponds a pole of $\hat{\rho}(s)$ in the right-half plane symmetrically situated with respect to the $j\omega$-axis and vice versa. We also assume here that if $D(s^2)$ has any zero at origin, it is

*Henceforth we will consider only normalized $Z(s)$ and $\hat{Z}(s)$.

not cancelled with the factor s in the numerator of $\hat{Z}(s)$. Moreover, since the algebraic sum of an even and an odd polynomial with real coefficients cannot have any zeros on the jω-axis unless each of them have the same jω-axis zeros, it follows that $\hat{\rho}(s)$ will not have any jω-axis poles and zeros except possibly at the origin. Thus we can write*

$$\hat{\rho}(s) = \frac{m_1 - n_1}{m_1 + n_1} \cdot \frac{m_2 + n_2}{m_2 - n_2} \tag{4}$$

where $m_1 + n_1$ is the unique Hurwitz polynomial $H_1(s)$ formed by factoring the left-half plane poles of $\hat{\rho}(s)$ and $m_2 + n_2$ is the unique Hurwitz polynomial $H_2(s)$ obtained by factoring the left-half plane zeros of $\hat{\rho}(s)$ including that at origin, if any. The reflection coefficient of an active driving-point impedance can be thought to consist of two factors, one representing completely a passive network and the other including the active portion. Thus following Kelly's[4] approach, we can factor $\hat{\rho}(s)$ as

$$\hat{\rho}(s) = E(s) \cdot \hat{\rho}_1(s) \tag{5}$$

where

$$E(s) = \frac{m_1 - n_1}{m_1 + n_1}, \tag{6}$$

$$\hat{\rho}_1(s) = \frac{m_2 + n_2}{m_2 - n_2} \tag{7}$$

Because of the construction procedure outlined above, we note that E(s) satisfies the properties of the reflection coefficient of a passive LC driving-point impedance, i.e., E(p), where $p = (1 - s)/(1 + s)$, is a "unit function"[6].

---

*The even part of the polynomials are denoted by the symbol m and the odd part by the symbol n.

From (5) and (2) we obtain [4]

$$\hat{Z}(s) = \frac{1 - \hat{\rho}(s)}{1 + \hat{\rho}(s)}$$

$$= \frac{1}{\hat{Z}_r(s) + \frac{1}{\hat{Z}_1(s)}} + \frac{1}{\hat{Z}_1(s) + \frac{1}{\hat{Z}_r(s)}} \qquad (8)$$

where

$$\hat{Z}_r(s) = \frac{1 - E(s)}{1 + E(s)} \qquad (9)$$

$$\hat{Z}_1(s) = \frac{1 - \hat{\rho}_1(s)}{1 + \hat{\rho}_1(s)}$$

Substituting the expressions for E(s) from (6) and $\hat{\rho}_1(s)$ from (7) in (9) we obtain

$$\hat{Z}_r(s) = \frac{n_1}{m_1} \qquad (10)$$

$$\hat{Z}_1(s) = \frac{n_2}{m_2}$$

$-\hat{Z}_1(s)$, $-\frac{1}{\hat{Z}_1(s)}$, $+\hat{Z}_r(s)$ and $+\frac{1}{\hat{Z}_r(s)}$ are guaranteed to be in the form of passive LC driving-point impedances because of the properties of the Hurwitz polynomials. [7] The corresponding circuit representation of (8) as proposed by Kelly [4] is as shown in Fig. 2.

By making an inverse LC-RC transformation, we obtain a realization for $Z(s)$ using two UNIC's[+] as the active devices, as shown in Fig. 3. $Z_a(s)$, $Z_b(s)$, $Z_c(s)$ and $Z_d(s)$ of Fig. 3 are related to $\hat{Z}_1(s)$ and $\hat{Z}_r(s)$ by the following relations:

$$Z_a(s) = \frac{1}{\sqrt{s}} \cdot \hat{Z}_r(\sqrt{s})$$

$$Z_b(s) = \frac{1}{\sqrt{s}} \cdot \frac{1}{\hat{Z}_r(\sqrt{s})} = \frac{1}{s\, Z_a(s)}$$

$$Z_c(s) = \frac{1}{\sqrt{s}} \cdot \hat{Z}_1(\sqrt{s}) \qquad (11)$$

$$Z_d(s) = \frac{1}{\sqrt{s}} \cdot \frac{1}{\hat{Z}_1(\sqrt{s})} = \frac{1}{s\, Z_c(s)}$$

It is clear that $Z_a(s)$, $Z_b(s)$, $Z_c(s)$ and $Z_d(s)$ are in the form of passive RC driving-point impedances.

B. Special Case

We will now show that if the given driving-point impedance $Z(s)$ is a passive RC impedance function, then no active elements will be present in the structure of Fig. 3. In this case $\hat{Z}(s)$, obtained by the RC-LC transformation of (1), is of the form of passive LC driving-point impedance. Furthermore, because of the indicated transformation procedure, the numerator of $\hat{Z}(s)$ will be an odd polynomial. Hence we can write

[+]A negative impedance converter with a unity conversion ratio will be designed as an UNIC.

$$\hat{Z}(s) = \frac{n_1}{m_1} \tag{12}$$

and the corresponding reflection coefficient is

$$\hat{\rho}(s) = \frac{m_1 - n_1}{m_1 + n_1} \tag{13}$$

Comparing (13) with (4) we note that (13) is a special case of (4) when $n_2 = 0$. This implies, in this special case, Fig. 2 and consequently Fig. 3 are modified to that of Fig. 4a and Fig. 4b, respectively, each of which, as can be seen, do not contain any UNIC.

C. The Decomposition

From (4) and (8) we obtain

$$\hat{Z}(s) = \frac{n_1 m_2 - m_1 n_2}{m_1 m_2 - n_1 n_2} \tag{14}$$

m_1, m_2 are even polynomials and n_1, n_2 are odd polynomials. Hence we can express them as

$$\begin{aligned} m_1 &= a_1(s^2) \\ m_2 &= a_2(s^2) \\ n_1 &= s\, b_1(s^2) \\ n_2 &= s\, b_2(s^2) \end{aligned} \tag{15}$$

Substituting (15) in (14) and then applying the inverse LC-RC transformation we arrive at the following theorem:

Theorem - Any real rational function of s, Z(s), can always be decomposed in the following form,

$$Z(s) = \frac{b_1(s)a_2(s) - a_1(s)b_2(s)}{a_1(s)a_2(s) - s\, b_1(s)b_2(s)} \tag{16}$$

where $a_1(s)/s\, b_1(s)$, $b_1(s)/a_1(s)$, $a_2(s)/s\, b_2(s)$ and $b_2(s)/a_2(s)$ are unique functions satisfying the properties of passive RC driving-point impedances.

Identification procedures for $a_1(s)$, etc., can be summarized as follows:

1. From the given $Z(s) = N(s)/D(s)$, obtain the polynomial $Q(s) = D(s^2) - s\, N(s^2)$,
2. Find the zeros of the polynomial $Q(s)$. Form the polynomial $m_2 + n_2$ by factoring the left-half plane zeros of $Q(s)$, including the zero at origin, if any, and factor out the right-half plane zeros of $Q(s)$ to form the polynomial $m_1 - n_1$,
3. From $m_2 + n_2$ and $m_1 - n_1$ obtain the even and odd parts and using (15) obtain $a_1(s)$, $a_2(s)$, $b_1(s)$ and $b_2(s)$.

D. The Associated Function

Before going into the application of (16) in the synthesis of active RC networks, we will present an interpretation of (14). Let us define the function

$$F(s) = \frac{m_1+n_1}{m_2+n_2} \tag{17}$$

as the associated function corresponding to Z(s). Though $m_1 + n_1$ and $m_2 + n_2$ are Hurwitz polynomials, F(s) may not be a p.r. function. The odd and even parts of F(s) are

$$\text{Od } F(s) = \frac{n_1m_2 - m_1n_2}{m_2^2 - n_2^2}$$

$$\text{Ev } F(s) = \frac{m_1m_2 - n_1n_2}{m_2^2 - n_2^2} \tag{18}$$

Thus we can write

$$s\ Z(s^2) = \hat{Z}(s) = \frac{\text{Od } F(s)}{\text{Ev } F(s)} \tag{19}$$

The above results can be summarized as:

Theorem - To every real rational function $Z(s)$, there corresponds a unique associated function $F(s)$ defined as a ratio of two unique Hurwitz polynomials and related to $Z(s)$ according to (19).

Now $D(s^2)$ corresponds to the numerator of the even part of $F(s)$. If $D(s^2)$ is positive on the $j\omega$-axis, then $F(s)$ will be a p.r. function. In case of $D(s^2)$ having odd ordered $j\omega$-axis zeros, $D(-\omega^2)$ will not be positive for all ω's. In that case we can multiply both the numerator and the denominator of $s\ Z(s^2)$ by the factor $\prod_i(s^2 + \omega_i^2)$ so that the augmented $D(s^2)\prod_i(s^2 + \omega_i^2)$ has all even ordered $j\omega$-axis zeros and thus stays positive on the $j\omega$-axis. Then we can relate the modified $s\ Z(s^2)$ to a p.r. function $G(s)$ according to (20), which is given below.

$$s\ Z(s^2) = \frac{s\ N(s^2)\prod_i(s^2+\omega_i^2)}{D(s^2)\prod_i(s^2+\omega_i^2)} = \frac{\text{Od } G(s)}{\text{Ev } G(s)} \tag{20}$$

Thus we find that there also exists a p.r. function $G(s)$ corresponding to $Z(s)$.

III. SYNTHESIS PROCEDURES USING ONE ACTIVE ELEMENT

A. The Cascade Method[2]

The input impedance $Z(s)$ of the cascade structure of Fig. 5 is given as

$$Z(s) = z_{11}\frac{\frac{1}{y_{22}} - Z_L}{z_{22} - Z_L} \tag{21}$$

where z_{11}, z_{22} and y_{22} are the two-port parameters of the network N.

Rearranging (16) we obtain,

$$Z(s) = \frac{a_1(s)}{s\, b_1(s)} \cdot \frac{\dfrac{b_1(s)}{a_1(s)} - \dfrac{b_2(s)}{a_2(s)}}{\dfrac{a_1(s)}{s\, b_1(s)} - \dfrac{b_2(s)}{a_2(s)}} \tag{22}$$

Comparing (21) and (22) we identify

$$z_{11} = z_{22} = \frac{a_1(s)}{s\, b_1(s)}$$

$$\frac{1}{y_{22}} = \frac{b_1(s)}{a_1(s)} \tag{23}$$

$$Z_L = \frac{b_2(s)}{a_2(s)}$$

From (23) we obtain

$$z_{12}^2 = \frac{a_1^2(s) - s\, b_1^2(s)}{s^2\, b_1^2(s)} \tag{24}$$

For the set of parameters $\{z_{ij}\}$ of (23) and (24) to represent a physically realizable set, the residue condition must be satisfied. Evaluating the residues of z_{11}, z_{22} and z_{12} in a pole at $s = s_k$, we obtain

$$k_{11} = k_{22} = \left(\frac{a_1(s)}{b_1(s) + s\, \dot{b}_1(s)}\right)\Bigg|_{s = s_k}$$

$$k_{12} = \pm\left(\frac{\sqrt{a_1^2(s) - s\, b_1^2(s)}}{b_1(s) + s\, \dot{b}_1(s)}\right)\Bigg|_{s = s_k} = \pm\left(\frac{a_1(s)}{b_1(s) + s\, \dot{b}_1(s)}\right)\Bigg|_{s = s_k} \tag{25}$$

where $\dot{b}_1(s)$ is the derivative of $b_1(s)$. It is seen that the residue condition is satisfied with an equal sign.

Next, we will have to show that a rational z_{12} can be obtained. In general, the numerator of z_{12}^2 as given in (24) will not be a perfect square. If it is not, the following augmentation method which is similar to Darlington's technique [8] can be pursued:

Augment the <u>associated</u> function F(s) by a Hurwitz polynomial $m_0 + n_0$ to obtain the modified function

$$F(s) = \frac{m_1+n_1}{m_2+n_2} \cdot \frac{m_0+n_0}{m_0+n_0} = \frac{m_1'+n_1'}{m_2'+n_2'} \tag{26}$$

where

$$\begin{aligned} m_1' &= m_1m_0 + n_1n_0 \\ m_2' &= m_2m_0 + n_2n_0 \\ n_1' &= n_1m_0 + n_0m_1 \\ n_2' &= n_2m_0 + n_0m_2 \end{aligned} \tag{27}$$

Recomputing the odd and the even parts of the augmented <u>associated</u> function we can show that there is no change in the ratio of the odd part to the even part, both of them being multiplied by the common factor $m_0^2 - n_0^2$. But after augmentation the two-port parameters and Z_L of (23) and (24) will be modified as follows:

$$z_{11} = z_{22} = \frac{a_1(s)a_0(s) + s\, b_1(s)b_0(s)}{s\, b_1(s)a_0(s) + s\, a_1(s)b_0(s)}$$

$$z_{12} = \pm \frac{\sqrt{\left\{a_1^2(s) - s\, b_1^2(s)\right\}\left\{a_0^2(s) - s\, b_0^2(s)\right\}}}{s\, b_1(s)a_0(s) + s\, a_1(s)b_0(s)} \tag{28}$$

$$Z_L = \frac{b_2(s)a_0(s) + b_0(s)a_2(s)}{a_2(s)a_0(s) + s\, b_2(s)b_0(s)}$$

We can choose $a_0^2(s) - s\,b_0^2(s)$ equal to that factor of $a_1^2(s) - s\,b_1^2(s)$ which is not a perfect square and thus obtain a rational z_{12}. It can be shown that the residue condition is still satisfied with an equal sign.

Furthermore, the decomposition technique guarantees z_{11}, z_{22}, $1/y_{22}$ and Z_L to be of the form of passive RC driving-point impedances.

A lattice realization is always possible because of the residues of the z-parameters being equal in magnitude. Thus any real rational Z(s) can be realized in the practical Cascade Configuration of Fig. 5 and, in effect, we have proved the theorem mentioned in the Introduction.

B. Synthesis Using Special Structures[3]

The problem of obtaining a rational z_{12} associated with the Cascade Method is not encountered in the following method. This method uses only one-port passive RC networks and a generalized type of impedance converter* in special configurations. The h-matrix of a GIC is given by

$$\begin{bmatrix} 0 & 1 \\ \dfrac{Z_m}{Z_n} & 0 \end{bmatrix} \tag{29}$$

where Z_m and Z_n are passive RC driving-point impedances.

Consider the network of Fig. 6, which we will designate as Type I - Special Configuration. The input impedance of this network is

$$Z(s) = \frac{Z_4 - Z_3}{\dfrac{Z_4}{Z_1} - \dfrac{Z_3}{Z_2}} \tag{30}$$

Rearranging (16) we obtain

$$Z(s) = \frac{\dfrac{b_1(s)}{a_1(s)} - \dfrac{b_2(s)}{a_2(s)}}{1 - \dfrac{s\,b_1(s)}{a_1(s)} \cdot \dfrac{b_2(s)}{a_2(s)}} \tag{31}$$

*The Generalized Impedance Converter of this section will be designated as GIC. For idealized circuits and a transistor realization of GIC, see Ref. 3.

Comparing (30) and (31) we identify

$$Z_4 = \frac{b_1(s)}{a_1(s)} \quad ; \qquad Z_3 = \frac{b_2(s)}{a_2(s)}$$

$$Z_1 = \frac{b_1(s)}{a_1(s)} \quad ; \qquad Z_2 = \frac{a_1(s)}{s\, b_1(s)} \tag{32}$$

Because of the decomposition technique the set of impedances Z_1, Z_2, Z_3 and Z_4 are in the form of passive RC driving-point impedances.

The input impedance Z(s) of the Type II - Special Configuration of Fig. 7 is given as

$$Z(s) = \frac{Z_8 Z_6 - Z_7 Z_5}{Z_8 - Z_7} = \frac{Z_6 - \dfrac{Z_7 \cdot Z_5}{Z_8}}{1 - \dfrac{Z_7}{Z_8}} \tag{33}$$

Comparing (33) with (31), we obtain

$$Z_7 = \frac{b_2(s)}{a_2(s)} \quad ; \qquad Z_8 = \frac{a_1(s)}{s\, b_1(s)}$$

$$Z_6 = \frac{b_1(s)}{a_1(s)} \quad ; \qquad Z_5 = \frac{a_1(s)}{s\, b_1(s)} \tag{34}$$

It is seen that the impedances given in (34) are all passive RC driving-point impedance functions.

IV. ILLUSTRATIONS

A. Example 1

Consider the realization of an inductance by an active RC network. Let

$$Z(s) = s \tag{35}$$

We thus have

$$N(s) = s$$
$$D(s) = 1$$

Factorizing

$$Q(s) = D(s^2) - s\,N(s^2) = 1 - s^3$$

we obtain

$$Q(s) = (1 - s)(s^2 + s + 1)$$

Hence we identify

$$m_2 + n_2 = s^2 + s + 1$$
$$m_1 - n_1 = 1 - s$$

that means

$$m_1 = 1;\ n_1 = s$$
$$m_2 = s^2 + 1;\ n_2 = s$$

From this we obtain

$$a_1(s) = 1;\ b_1(s) = 1$$
$$a_2(s) = s + 1;\ b_2(s) = 1 \qquad (36)$$

The Cascade Realization - Substituting (36) in (24) we find that

$$z_{12} = \pm \frac{\sqrt{1 - s}}{s}$$

is not a rational function. We thus choose

$$a_0^2(s) - s\,b_0^2(s) = 1 - s$$

i.e.,

$$a_0(s) = 1;\ b_0(s) = 1 \qquad (37)$$

Finally, from (36), (37) and (28) we obtain the modified z-parameters and Z_L as

$$z_{11} = z_{22} = \frac{1+s}{2s}$$

$$z_{12} = \pm \frac{1-s}{2s} \qquad (38)$$

$$Z_L = \frac{s+2}{2s+1}$$

The complete network considering the positive sign for z_{12} is as shown in Fig. 8.

Type I Realization - From (36) and (32) we obtain

$$Z_4 = 1; \quad Z_3 = \frac{1}{s+1}$$

$$Z_1 = 1; \quad Z_2 = \frac{1}{s}$$

and the corresponding network is shown in Fig. 9.

Type II Realization - Substitution of (36) and (34) results in

$$Z_7 = \frac{1}{s+1}; \quad Z_8 = \frac{1}{s}$$

$$Z_6 = 1 \quad ; \quad Z_5 = \frac{1}{s}$$

and the corresponding network is shown in Fig. 11.

B. Example 2

Consider the realization of a negative inductance. It should be noted from (16) that in case of $-Z(s)$, $a_1(s)$ and $a_2(s)$, and $b_1(s)$ and $b_2(s)$, as obtained for $+Z(s)$, are interchanged, respectively. As a result, we obtain for this example from (36) the following:

$$a_1(s) = s + 1 \; ; \; b_1(s) = 1$$

$$a_2(s) = 1 \quad ; \; b_2(s) = 1 \qquad (39)$$

Substituting (39) in (24) we obtain

$$z_{12} = \pm \frac{\sqrt{s^2 + s + 1}}{s}$$

Augmenting we obtain a rational z_{12} and modified z_{11}, z_{22} and Z_L as given below,

$$z_{11} = z_{22} = \frac{s^2 + 3s + 1}{2s(s+1)} = \left[\frac{1}{2} + \frac{1}{2(s+1)}\right] + \left[\frac{1}{2s}\right]$$

$$z_{12} = \frac{s^2 + s + 1}{2s(s+1)} = \left[\frac{1}{2} - \frac{1}{2(s+1)}\right] + \left[\frac{1}{2s}\right] \qquad (40)$$

$$Z_L = \frac{s+2}{2s+1}$$

and the final network obtained considering the plus sign for z_{12} is as shown in Fig. 12.

Instead of realizing the z-parameters in a lattice structure, it is possible to obtain an unbalanced structure by means of the Fialkow-Gerst method.[9] For the given example, the parameters were partitioned as shown by the dotted lines in (40) and the resulting unbalanced structure is shown in Fig. 12.

This example was presented to illustrate the possibility of obtaining unbalanced structures and also to point out that the suggested method does not always lead to active RC networks with the least number of elements because of the well known fact that a negative inductance can be realized by terminating a negative impedance inverter by a positive capacitance.

C. Example 3

As a final example we will realize an impedance function whose numerator and denominator degrees differ by two. Let

$$Z(s) = \frac{1}{s^2 + 2s + 2} \qquad (41)$$

Following the procedure outlined in Example 1, we obtain

$$a_1(s) = s + 1;\ b_1(s) = 1$$
$$a_2(s) = s + 2;\ b_2(s) = 1 \qquad (42)$$

and it can be seen that for a cascade realization augmentation is necessary to insure a rational z_{12}. Finally, we obtain a network realization as shown in Fig. 13, using the formulas of (28).

V. CONCLUSION

It has been demonstrated that any real rational function of the complex variable can be always decomposed in a unique form and can be realized as the driving-point impedance of a one-port transformerless active RC network containing only one negative impedance converter. The suggested realization networks are practical and the element values of the associated[1] RC networks are easily obtained. Since the main part of the method consists of solving a linear equation for its real and complex roots, the complete method can be programmed easily for a digital computer.

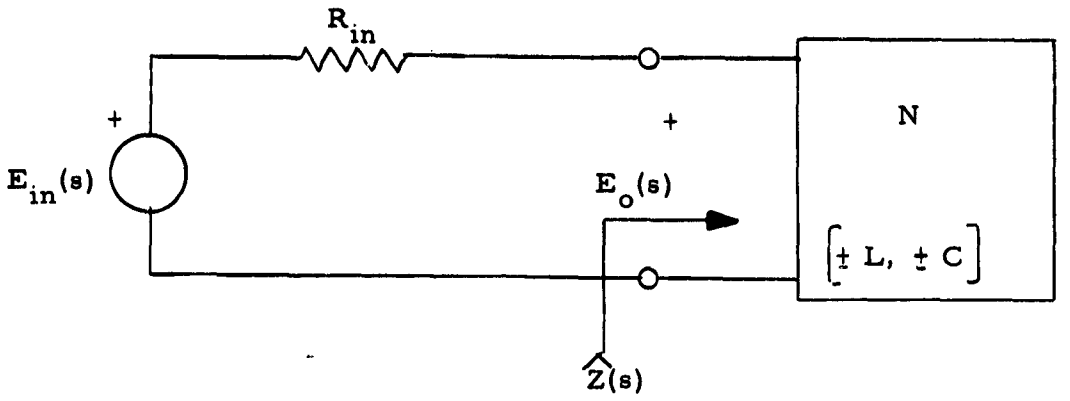


Figure 1

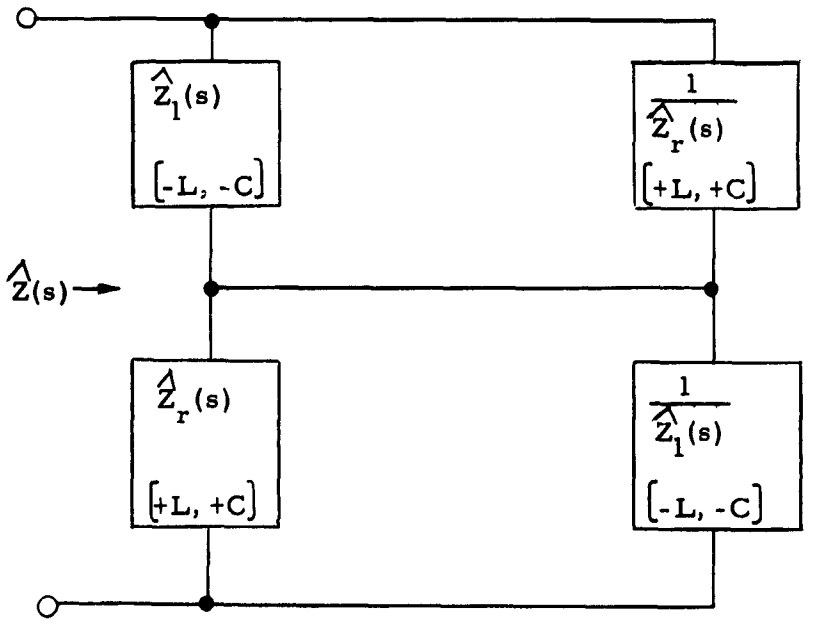
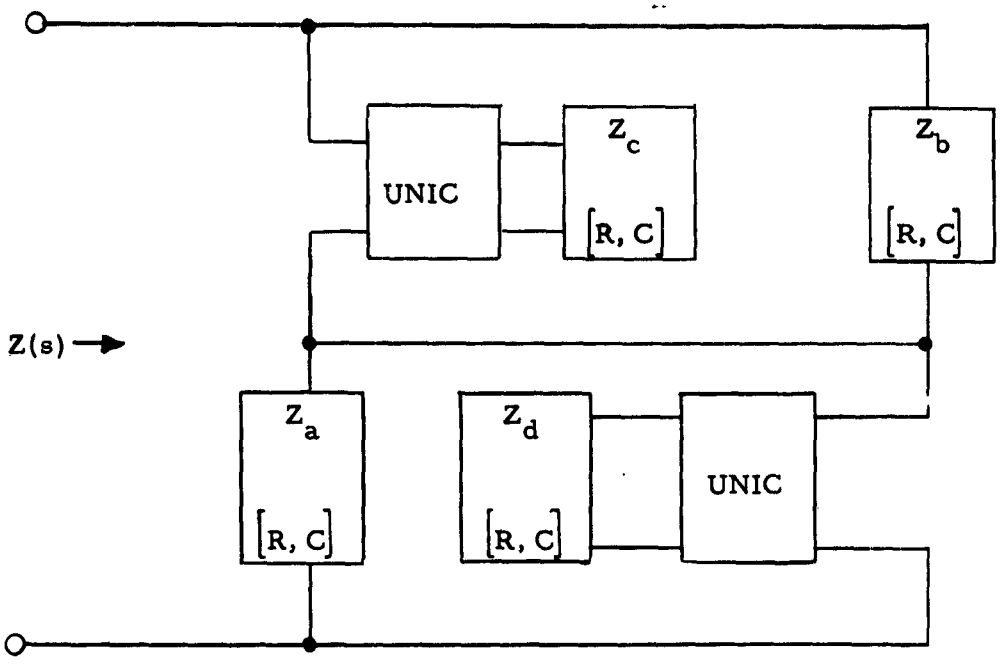


Figure 2



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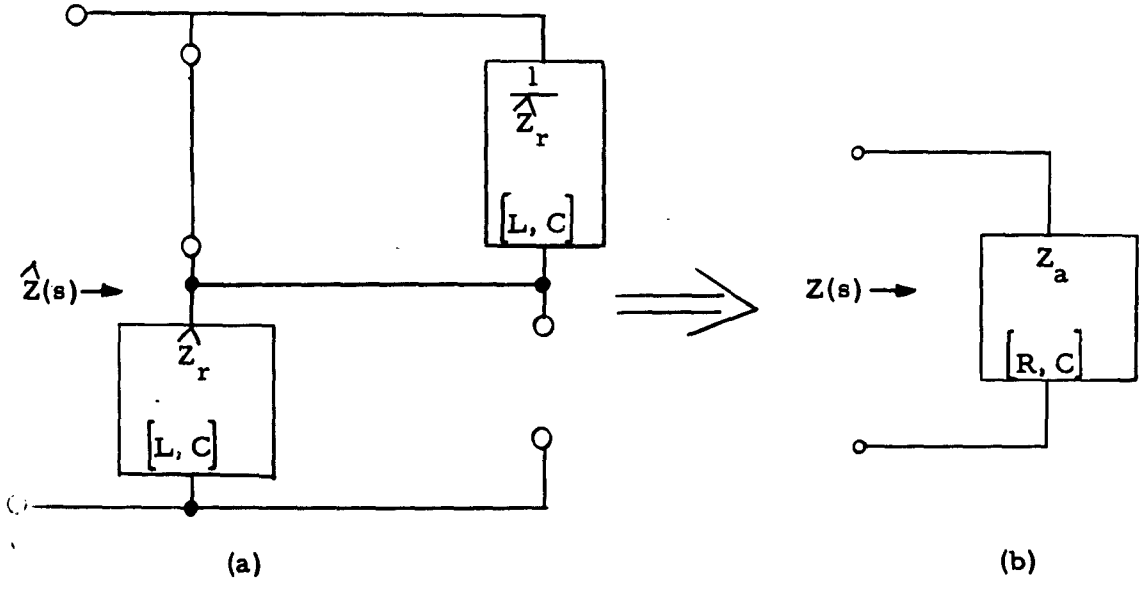

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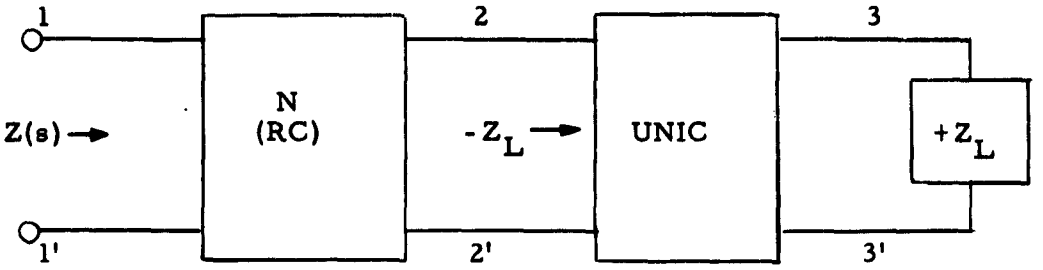

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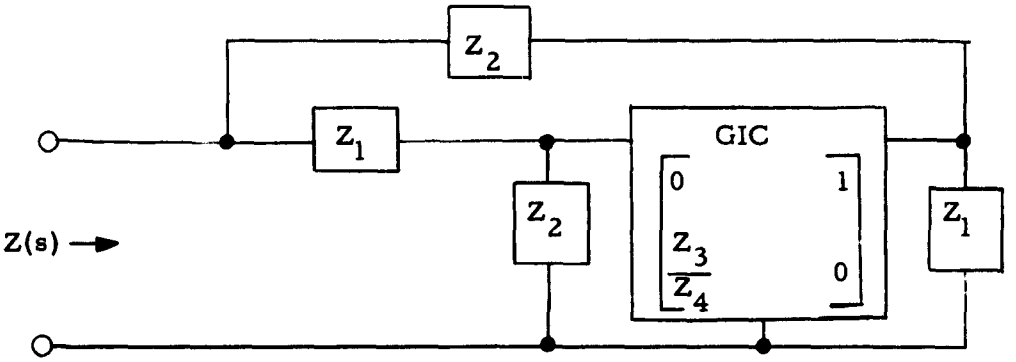

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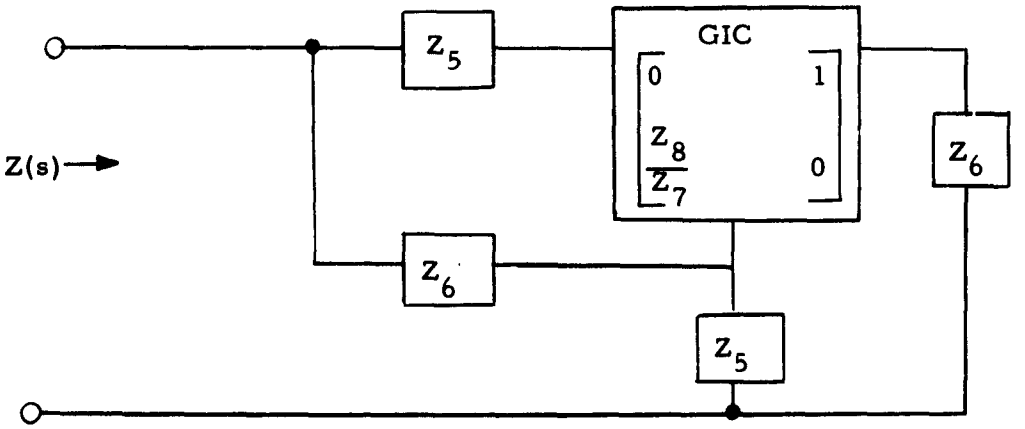

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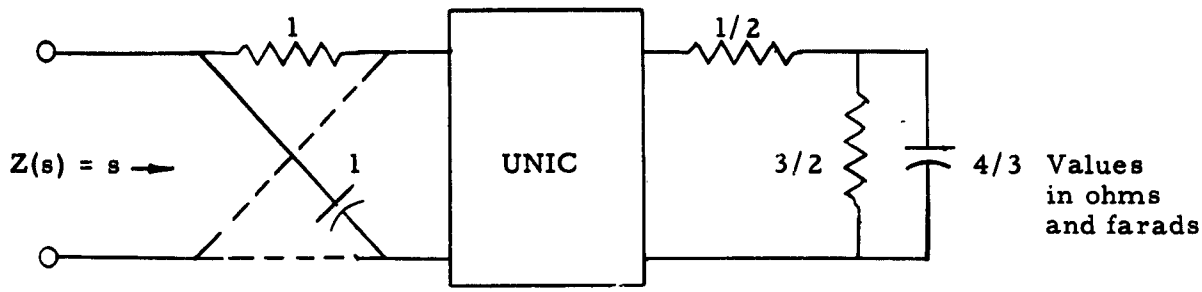

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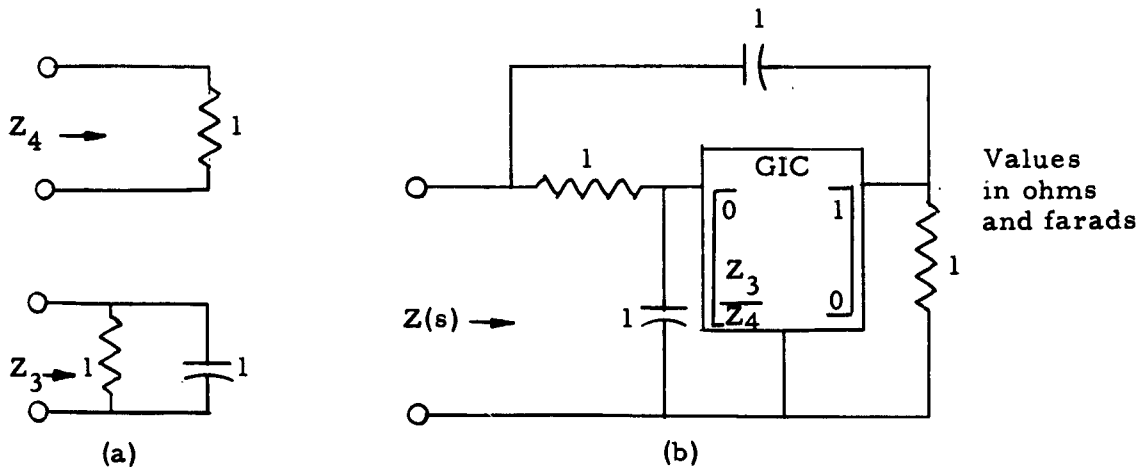

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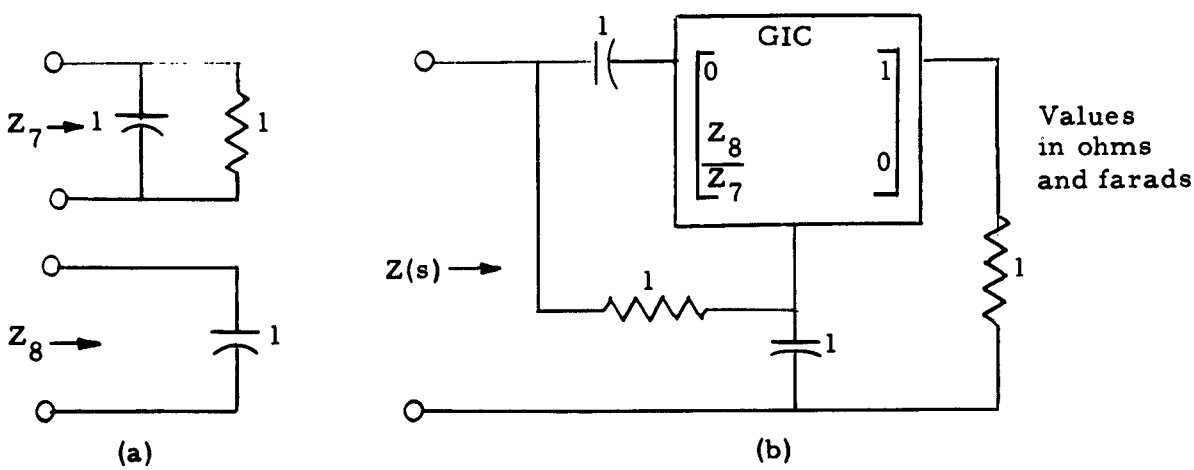

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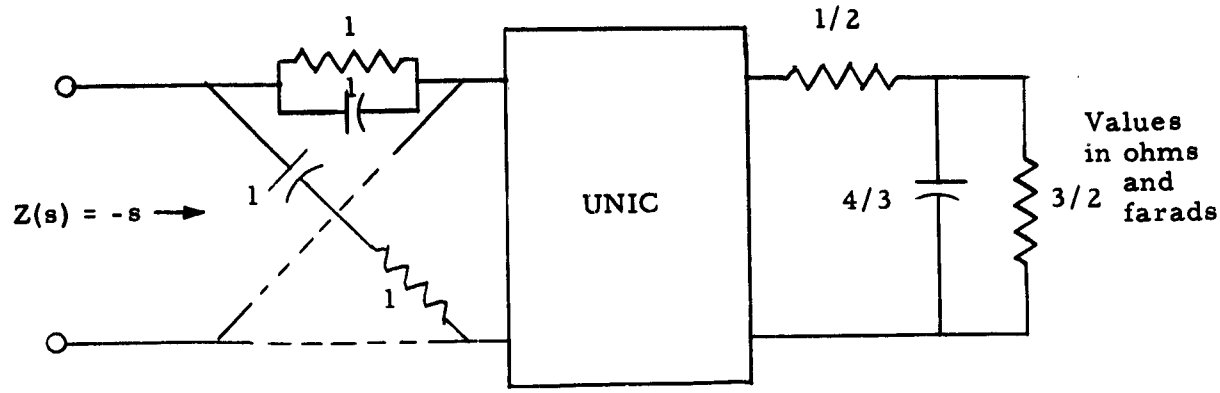

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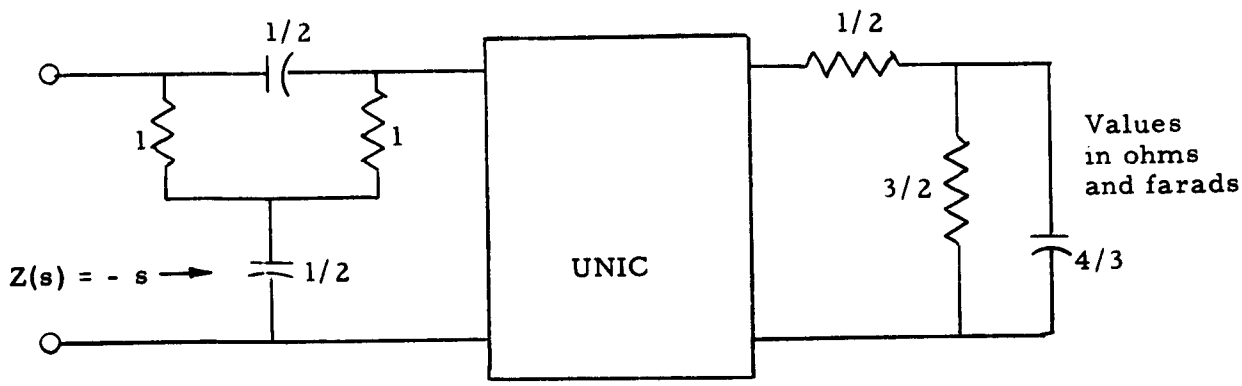

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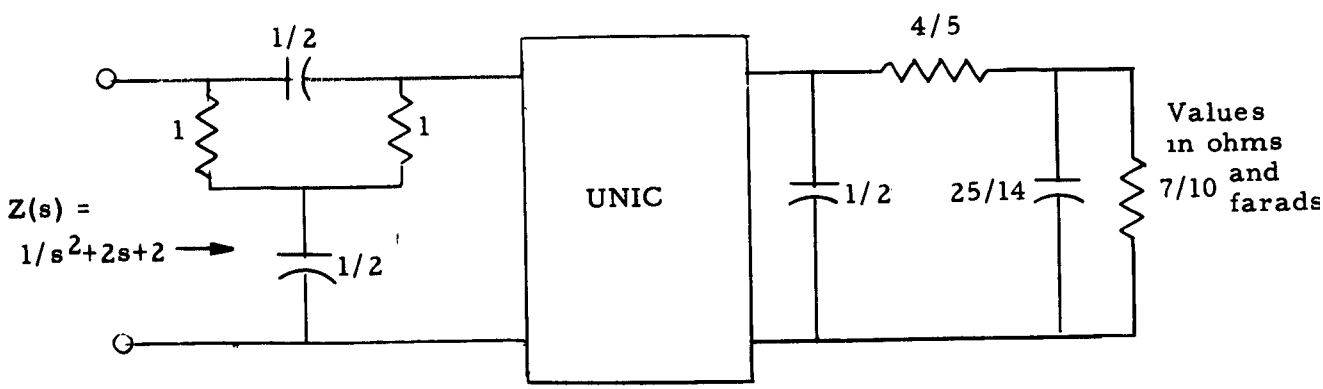

Figure 13

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